

## Topic Study Group 9

### Teaching and Learning of Geometry at Secondary Level

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**ABSTRACT** At ICME-14, the Topic Study Group (TSG) on the teaching and learning of geometry at the secondary school level, TSG-9, enabled participants from around the world to share research results, research projects, new developments, and updates on ongoing projects concerning geometry education at the secondary school level. The TSG embraced the four themes of connections between geometry education and mathematical practices and processes, teacher preparation and teacher knowledge for geometry, developments in geometry teaching, and curricular issues in school geometry. The discussion during the TSG sessions at the congress benefitted from the good range of quality presentations on each of the themes.

*Keywords:* Teaching; Learning; Geometry; Secondary school.

#### 1. TSG-9 Themes and Description

Geometry holds a major place within the secondary school mathematics curriculum. In preparing for the Topic Study Group (TSG) on the teaching and learning of geometry at the secondary school level, TSG-9, at ICME-14, the TSG-9 Team built on existing research and development on geometry education, including the equivalent TSG at ICME-13 (Herbst, Cheah, Richard and Jones, 2018) and reviews such as Jones and Tzekaki (2016), to identify a set of themes to guide and encourage contributions and discussions. The themes were as follows:

- Connections between secondary school geometry education and mathematical practices and processes such as argumentation and proof, visualization, figuration, and instrumentation;

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- Teacher preparation and teacher knowledge for geometry at the secondary school level;
- Developments in secondary school geometry teaching, including geometrical modeling and out-of-school problem solving;
- Curricular issues in secondary school geometry, including reform initiatives in school geometry, and new forms and applications of geometry.

In advance of ICME-14, TSG-9 received 43 submissions from all around the world, thereby providing diverse cultural representation that addressed the full range of the identified themes. Of the submitting participants, a number were impacted by the Covid19 pandemic prevalent at the time and could not take part in the TSG at ICME-14. During the congress, 23 papers and 4 posters were presented in the final programme for TSG-9.

## 2. TSG-9 Programme

During the TSG-9 sessions, one part of the programme of presentations and discussion was devoted to each of the themes. Within three of the four themes, the TSG-9 Team identified one paper for extended presentation to enable deeper discussion. Each themed session was chaired by a member of the TSG-9 Team.

### 2.1. *Geometry education and mathematical practices and processes*

The theme of connections between secondary school geometry education and mathematical practices and processes (such as argumentation and proof, visualization, figuration, and instrumentation) comprised the four papers set out in Tab. 1.

Tab. 1. Geometry education and mathematical practices and processes

<b>Paper and author(s)</b>	
[1]	A teacher's use of dynamic digital technology to address students' misconceptions concerning the use of additive strategies within geometric similarity. <i>Ali Simsek, Celia Hoyles, and Alison Clark-Wilson</i> (UK).
[2]	Students spatial ability and solving-strategies for spatial geometrical, mathematical, and physical task. <i>Marion Zoeggeler and Guenter J. Maresch</i> (Austria).
[3]	Introduction of an auxiliary element as a shift of attention. <i>Alik Palatnik and Avi Sigler</i> (Israel).
[4]	Construction program as a link between drawing and language to prepare proof process. <i>Joris Mithalal</i> (France).

Within this theme chaired by Joris Mithalal, the paper for extended presentation was by Alik Palatnik.

## 2.2. Teacher preparation and teacher knowledge for geometry

The theme of teacher preparation and teacher knowledge for geometry at the secondary school level comprised the four papers set out in Tab. 2.

Tab. 2. Teacher preparation and teacher knowledge for geometry

Paper and author(s)	
[5]	Understanding student teachers' mathematical knowledge for teaching geometry in a history of mathematics course. <b>Svein Arne Sikko, Iveta Kohanová, Magdalini Lada, and Liping Ding</b> (Norway).
[6]	Teacher knowledge related to secondary school level geometry: Evidence from teacher development in South Africa. <b>Jogymol Alex</b> (South Africa).
[7]	A pre-service teacher mental structure development for understanding the geometric reflection in terms of motion and mapping view: Alexis case. <b>Murat Akarsu</b> (Turkey)
[8]	Distinguishing content knowledge and pedagogical content knowledge for geometry teaching. <b>Liping Ding and Keith Jones</b> (Norway).

Within this theme chaired by Liping Ding, each presentation was allotted the same time.

## 2.3. Developments in geometry teaching

The theme of developments in secondary school geometry teaching (including geometrical modeling and out-of-school problem solving) comprised the eight papers set out in Tab. 3.

Tab. 3. Developments in geometry teaching

Paper and author(s)	
[9]	Possibility of the pirates' treasure problem for teaching elementary geometry. Satoshi Takahashi, <b>Ryoto Hakamata and Koji Otaki</b> (Japan).
[10]	Inquiry-based learning using the centroids of the circumscribed equilateral triangles. <b>Yuki Osawa</b> (Japan)
[11]	Study of angles and trigonometric ratio for 7th grade. <b>Tsuyoshi Sonoda</b> (Japan).
[12]	Decomposing proof in secondary classrooms: A promising intervention for school geometry. <b>Michelle Cirillo</b> (USA).
[13]	Distance under the magnifying glass: Developing series of problems around fundamental concepts in geometry. <b>Eszter Varga</b> (Hungary).
[14]	The grasp of the Pythagorean Theorem and its proof by Chinese pre-service mathematics teachers. <b>Hai Li</b> (China).
[15]	Implicative relationships among spatial perception, mental rotation and spatial visualisation: Implications for teaching geometry. <b>Melih Turgut and Iveta Kohanová</b> (Norway).
[16]	Geometry modelling outdoors with MATHCITYMAP. <b>Matthias Ludwig, Iwan Gurjanow, Simone Jablonski, and Moritz Baumann-Wehner</b> (Germany).

Within this theme chaired by Matthias Ludwig, the paper for extended presentation was by Michelle Cirillo.

#### 2.4. Curricular issues in secondary school geometry

The theme of curricular issues in secondary school geometry (including reform initiatives in school geometry, and new forms and applications of geometry) comprised the seven papers set out in Tab. 4.

Tab. 4. Curricular issues in secondary school geometry

Paper and author(s)
[17] Online formative assessment in geometry proving. <i>Yael Luz and Michal Yerushalmy</i> (Israel).
[18] Geometric reasoning and mechanics experiment: A case study of interdisciplinary integration teaching with graphic center of gravity as an example. <i>Feishi Gu, Zhenzhen He, and Liya Ban</i> (China).
[19] A study on the performance of seventh-grade students in mathematical spatial reasoning. <i>Zhikun Zhang and Jian Liu</i> (China).
[20] Didactic suitability characterization of three levels of achievement on geometric drawing of secondary school students. <i>Javier Díez-Palomar and Elvira García-Mora</i> (Spain).
[21] Let's make a circle by three persons. <i>Ken-ichi Iwase</i> (Japan).
[22] Reconfiguration of polygons to determine the measurement of their area. <i>Melissa Denisse Castillo Medrano and Jesus Victoria Flores Salazar</i> (Peru).
[23] High school learners' preconceptions on the classification of quadrilaterals. <i>Judah Paul Makonye</i> (South Africa).

Within this theme chaired by Keith Jones, the paper for extended presentation was by Yael Luz.

### 3. Summary Discussion and Future Directions

Topic Study Group 9 (TSG-9) at ICME-14 brought together participants from around the world to share research results, research projects, new developments, and updates on ongoing projects concerning geometry education at the secondary school level. The discussion at TSG-9 benefitted from the good range of quality presentations on each of the themes.

The discussion during the theme of connections between secondary school geometry education and mathematical practices and processes ranged from considering the role of digital technologies in supporting this connection to the place of spatial reasoning and geometrical construction. Future directions are likely to continue to be on argumentation and proof in school geometry, and on visualization, figuration, and instrumentation processes.

During the theme of teacher preparation and teacher knowledge for geometry at the secondary school level, discussions focused primarily on teacher knowledge for secondary school geometry teaching. Future directions are likely to continue to be on such teacher knowledge and on the design of geometry education teacher development both pre-service and in-service.

The theme of developments in secondary school geometry teaching was wide ranging and addressed geometry teaching concerns from spatial perception and

visualisation to geometric proof. The presentation of teaching ideas was a strong and beneficial element. Future directions are likely to continue to be on the teaching of spatial and geometrical reasoning, along with geometrical modeling and out-of-school problem solving.

The theme of curricular issues in secondary school geometry was equally wide-ranging, with discussions enriched with ideas being used in schools alongside research into the secondary school geometry curriculum and its assessment. Future directions are likely to continue to be on the scope of the secondary school geometry curriculum, including reform initiatives and possible ways of incorporating new forms and applications of geometry that occur in mathematics.

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